the instantaneous submersion dose rate for semi-spherical infinite cloud by $\mu_{en}r$ (r is variable radius). Calculation of dose is done via integration over the time interval of interest; the integration and the final expression for dose which results are frequently considerably simpler than would be obtained if equation (1) were used to represent f in this calculation.

It should be noted that neither the value of f nor that of f' calculated here accounts for the effects of the ground surface on the dose, and some deviations from the calculated dose would be expected as a consequence of the air/ground inhomogeneity. In addition if a finite cloud is confined by solid walls the calculated dose will be low to the extent that photons scattered from the walls contribute to the dose. Also it should be kept in mind that the above discussion and development pertains only to the considerations of a tissue surface dose; the variation of dose rate as a function of depth in the body is a reality which may have to be accounted for, particularly in the case of low energy photons.

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Health Physics Pergamon Press 1974. Vol. 27 (July), pp. 155–157. Printed in Northern Ireland

A General Equation for the Kinetics of Linear First Order Phenomena and Suggested Applications

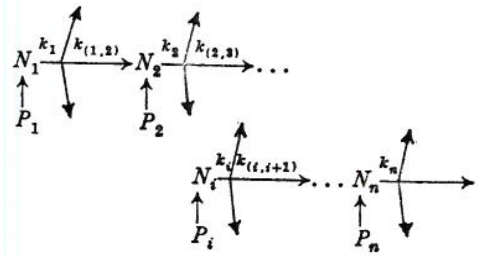
(*Received* 16 *August* 1973; *accepted* 14 *January* 1974)

Systems Described by Linear First Order Kinetics

THERE are many systems of concern in health physics in which quantities of interest are described by linear first order kinetics. Often an individual quantity is genetically related in a series to parent and daughter quantities through linear first order production and destruction phenomena. Differential equations describing the rate of production and destruction of individual members of such series may be solved by standard techniques to obtain quantities of interest. General equations have been obtained for some specific cases, including the familiar case for the serial transformation by radioactive decay of each member of a radioactive series as described by the Bateman equation, "BATEMAN (1910)" and others, "HULTQUIST (1956)" and "ICRP (1959)". The simple general equation given below may be applied to these as well as any system obeying linear first order kinetics, for example, chemical reactions. The equation gives the quantity of any member of a serial transformation allowing for any linear first order production and destruction phenomena, initial values for all members, constant independent production rates for all members, and unlimited branching. We have found the equation to be a very useful tool for describing (1) the serial transformation of radionuclides, (2) the inventory of quantities in the core of a nuclear reactor such as fission products, poisons, or fissile elements accounting for neutron burnup of all species (3) the collection and analysis of serially related radioactive aerosols such as radon, thoron, xenon or krypton daughters, (4) burden of radionuclides in various organs of the body and resulting doses for single intakes, multiple intakes, or continuous intakes of parent as well as daughter radionuclides, and (5) the quantity of airborne radioactive aerosols generated in a ventilated space through the decay of parent inert gases such as radon, thoron, xenon and krypton.

General Equation for the Kinetics of Linear First Order Phenomena

Consider the serial transformation by any linear first order process of each member in the series:



where

N_i = quantity of the i^{th} species present at a particular time, t,

k_i = total removal constant for the i^{th} species (i.e. the instantaneous fraction of the i^{th} species destroyed per unit time by all linear first order removal processes),

$k_{(i,i+1)}$ = partial removal constant for the i^{th} species (i.e. the instantaneous fraction of the i^{th} species transformed per unit time to the $(i+1)^{th}$ species), and related to the branching fraction, $f_{(i,i+1)}$:

$k_{(i,i+1)} = f_{(i,i+1)}k_i$,

and

P_i = constant independent rate of production of the i^{th} species.

The differential equations for the instantaneous time rates of change in the quantities of each member of the chain are:

$$\frac{dN_1}{dt} = P_1 - k_1N_1$$

$$\frac{dN_2}{dt} = P_2 + k_{(1,2)}N_1 - k_2N_2$$

$$\vdots$$

$$\frac{dN_i}{dt} = P_i + k_{(i-1,i)}N_{(i-1)} - k_iN_i,$$

$$\vdots$$

$$\frac{dN_n}{dt} = P_n + k_{(n-1,n)}N_{(n-1)} - k_nN_n.$$

These equations may be solved by standard methods to obtain the quantity of any member of the series. The general equation for the quantity of the n^{th} member of the series is given by

$$N_n = \sum_{i=1}^{i=n}\left[\left(\prod_{j=i}^{j=n-1} k_{(j,j+1)}\right) \times \sum_{j=i}^{j=n}\left(\frac{N_i^0 e^{-k_j t}}{\prod_{\substack{p=i\\ p\neq j}}^{p=n}(k_p - k_j)} + \frac{P_i(1 - e^{-k_j t})}{k_j \prod_{\substack{p=i\\ p\neq j}}^{p=n}(k_p - k_j)}\right)\right] \quad (1)$$

where

N_i^0 = quantity of i^{th} species present at some arbitrary reference time zero, and

t = generation or elapsed time.

Equation (1) embodies any first order process in which individual quantities are produced at constant independent rates and transformed at rates depending on the values of the quantities present and the first order rate constants. Any of the quantities from 1 to n may or may not be radioactive. Is the case of a system where quantities are genetically related only through radioactive transformations, the partial and total rate constants in equation (1) are appropriately the partial and total disintegration constants.

The simplicity of equation (1) may be appreciated immediately if one considers the familiar simple cases for the decay of a single radionuclide and the buildup of a single radionuclides produced at a constant rate. Equations for these cases are obtained for the parent of the chain for $n = 1$; $N_1 = N_1^0 e^{-\lambda_1 t}$ for $P_1 = 0$ and $N_1 = P_1(1 - e^{-\lambda_1 t})/\lambda_1$ for $N_1^0 = 0$, which are obvious from the two major terms in the inner summation.

Convergent and Divergent Branches

If a given quantity in a series is produced by a first order process from some branching chain, then by application of equation (1) over all applicable chains, it is possible to obtain the total value of this quantity by simple addition of values calculated from the various chains. However, in calculating the quantity of the n^{th} species from various contributing chains, the last term in the major summation, which is calculated for $i = n$, should not be added more than once since it represents contribution of the n^{th} species to itself. Similar considerations apply to any species following the n^{th} species. Divergent branches can be treated independently to yield quantities of interest.

Calculation of Disintegrations

Equation (1) may be multiplied by the total decay constant of the n^{th} member of a series and integrated over any time interval to obtain (1) the number of disintegrations for a time interval during generation accounting for all removal and transformation mechanisms, (2) the number of disintegrations for a time interval post generation when all production rates are zero but accounting for all removal and transformation mechanisms, and (3) the number of disintegrations for a time interval post generation when all production rates are zero and accounting for only radioactive decay. A specific example for each of these cases is: (1) the calculation of the disintegrations of a radionuclide in some organ of interest in the body during continuous uptake of the radionuclide, (2) the calculation of the disintegrations of a radionuclide in some organ of interest in the body post uptake, and (3) the calculation of the disintegrations post collection on a filter sample of a radionuclide serially related to other radionuclides. The most general case is that involving the calculation of the disintegrations for a time interval during generation. The number of disintegrations, $D_n(t_1 \to t_2)$, of the n^{th} member of the series over a time interval from t_1 to t_2 during generation is given by

$$D_n(t_1 \to t_2) = \sum_{i=1}^{i=n}\left[\lambda_n\left(\prod_{j=i}^{j=n-1} k_{(j,j+1)}\right) \times \sum_{j=i}^{j=n}\left(\frac{N_i^0(e^{-k_j t_1} - e^{-k_j t_2})}{k_j \prod_{\substack{p=i\\ p\neq j}}^{p=n}(k_p - k_j)} + \frac{P_i(k_j(t_2 - t_1) + e^{-k_j t_2} - e^{-k_j t_1})}{k_j^2 \prod_{\substack{p=i\\ p\neq j}}^{p=n}(k_p - k_j)}\right)\right]. \quad (2)$$

Equation (2) is appropriate for case 1, disintegrations during generation. The appropriate equation for case 2, disintegrations over the time interval from τ_1 to τ_2 post generation accounting for all removal and transformation mechanisms, is obtained from equation (2) with the condition that P_i equals zero. In this case it is understood that each value of N_i^0 is the quantity present at the end of the previous generation cycle and that t_1 and t_2 define the time interval post generation, τ_1 to τ_2, in which the disintegrations are to be calculated. The appropriate equation for case 3, disintegrations post generation from τ_1 to τ_2 accounting for only radioactive transformations is obtained from the equation representative of case 2 by noting appropriately that the partial and total rate constants are the partial and total disintegrations constants.

Applications

Applications of equation (1) have been suggested above. We have found the equation to be quite useful. Once the particular series or chain and the appropriate first order rate constants are identified, one can immediately write the equation for the quantity of any member of the series avoiding tedious solutions of the differential equations describing the production and destruction of individual members.

In applying the equations, one quickly notes that the total removal rate constants must be distinctly different. For cases in which the rate constants of two or more species are equal, the equation reduces to an indeterminate form since certain factors in the product, $\underset{\substack{p=i\\p\neq j}}{\overset{p=n}{\pi}}(k_p - k_j)$, would reduce to zero. However, approximate solutions may be obtained from the equations directly; otherwise, equations that give exact solutions may be obtained.

Details regarding the use and suggested applications of equation (1) and equation (2) are available from the authors.

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